Source Amnesia

Andrea Wilson & Lones Smith

UW Madison

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What Is Source Amnesia?

- Source amnesia is the inability to remember where, when, or how one has learned knowledge that has been acquired and retained
- Source amnesia (SA) may be a pathological disorder: Jason Bourne
- In fact, we all experience it daily, since the bibliographic sources of our knowledge is rarely useful for us
- SA can be "sourced" to an experimental neuropsychology paper: Stacker et al (1984), "Retrieval without Recollection: An Experimental Analysis of Source Amnesia", *Journal of Verbal Learning and Verbal Behavior*

"Your Brain Lies To You"

by Sam Wang and Sandra Aamodt, NY Times, June 2008

- *False beliefs are everywhere:*
- *18% of Americans believe that the sun revolves around the earth*
- *the quirky way our brains store memories: Each time we recall [information], our brain writes it down again, and during this re-storage, it is also reprocessed. In time, the fact is separated from the context in which it was learned.*
- *You probably know that the capital of California is Sacramento, but you probably don't remember how you learned it.*
- *This phenomenon, known as source amnesia, can also lead people to forget whether a statement is true.*

“Your Brain Lies To You”

by Sam Wang and Sandra Aamodt, NY Times, June 2008

- *We all experience source amnesia, the tendency to forget where we heard something, and the problem gets worse as we get older.*
- *An unfortunate side effect is that repeating myths or rumors, even to debunk them, tends to plant the false idea even more firmly in people's heads. People remember that they've heard the idea, but don't remember that they've heard it was false.*

Background: Economics Without Information Frictions

- the literatures on rational learning and rational expectations (1960s-today) has explored how bayesian individuals can slowly accumulate existing information, and how private information is imprinted in prices and thereby dispersed
- Key properties of such models:
  1. *martingale* character of belief processes — the foundation of macro and finance (eg. Euler equations), via the law of iterated expectations
  2. *path independence* of learning — how one arrives at a belief is irrelevant
  3. *limits on variance* in beliefs and prices — critiqued by Shiller, 1981
  4. *homo economicus* without psychological quirks (confirmation bias, eg.)

Background: Information Frictions In Economics

- Bounded memory, such as the finite state Bayesian automata in Hellman and Cover (1970), and Wilson (2014)
  - eg. explains many psychological affects (eg. confirmation bias)
  - individuals have no idea about how they arrived at any particular belief
- Informational herding (e.g. Smith and Sorensen 2000):
  - Only a coarse garbling of past signals is observed, by seeing past actions choices.
  - Public beliefs are a martingale.

- By contrast, in *Source Amnesia,* we will assume that signal realizations can be fully recalled; rather, we erase their bibliographic information
- This is an independent information friction, with its own implications

Two Manifestations of Source Amnesia

Our project has two applications:

1. behavioral decision theory, starring the source amnesiac **Sam**
2. an equivalent fully rational trading story, starring a sequence of myopic traders **Samantha**

Two Manifestations of Source Amnesia

- For Sam the behavioral decision theorist, source amnesia is an *assumed behavioral friction* (that has been well-documented).
  - Sam is Bayesian, and learns from signals
  - When he sees a signal, he may not recall its source, nor whether (and how often) he's seen it before
- For traders Samantha, source amnesia is a market implication:
  - She can learn from the asset price, records of past transactions, and any signals she sees today.
  - She does not know whether today's information has already been traded on/incorporated into the price, and may not know whether past trades were sparked by old or new information

Two Informational Flavors of Source Amnesia

1. Time Series Source Amnesia:
    - a source is a piece of information, which may be sampled more than once, possibly with noise
    - all sources are equally informative, but you may not recall whether you've already seen today's information source, or know whether its information has already been incorporated into your beliefs
2. Cross-Sectional Source Amnesia
    - an information source is something like Fox News, or the New York Times
    - some sources may provide better-quality, or less biased, information than others

Motivating Example 1: Time Series Source Amnesia

- Two equally likely states of the world, $\theta \in \{G, B\}$
- signal realizations, g or b matches the state with chance q
- On days 1 and 2, Sam consults an information source and gets a signal realization each time
- Day 2 observation is a "fresh" draw (i.e. conditionally iid) with chance λ, and otherwise is "stale" (repeats the day-1 signal)
- After seeing signal g on day 1, how should Sam react to seeing g again on day 2, if he forgets the day 1 source?

Sam's Rational Response to Source Amnesia

- since the two signals "match" Sam thinks it more likely that the day 2 signal is "stale"
- updated belief:

$$\frac{\Pr(G|gg)}{\Pr(B|gg)} = \frac{\lambda q^2 + (1-\lambda)q}{\lambda(1-q)^2 + (1-\lambda)(1-q)}$$

Sam's Reaction vs. Full Information Response

- if information actually is fresh, Sam underreacts, fearing that he might be seeing old news repeated
- if information actually is stale, Sam overreacts, thinking it *could* be new
- with a longer time horizon, Sam may rationally underreact to information that seems too consistent, especially in phases when information is being generated more quickly than expected
- while if information is generated more slowly than expected, Sam will overreact to information that is largely recycled

Personal Application

- real-life example: I used to follow anti-GMO ads, or commercials for "foods where you can actually pronounce the ingredients"
- these largely recycled the same study, but I kept buying into it
- I was later surprised by "new" info, flagging these studies as flawed and non-scientific, and my beliefs quickly adjusted

I am Food Babe Squirrel

Let me tell you, these ingredients are so terrible that even *I* cannot pronounce them

Motivating Example 2: Cross-Sectional Source Amnesia

- two possible states of the world, $\theta \in \{G, B\}$
- two information sources, each yielding a conditionally iid binary signal about θ each day, with signal realizations, g and b
- source 1 is "high-quality":

$$\Pr(g|G) = \Pr(b|B) = q \text{ near } 1$$

- source 2 is "low-quality":

$$\Pr(g|G) = \Pr(b|B) = p \in \left(\frac{1}{2}, q\right)$$

Applications

- if Sam forgets which observations came from which sources, then low-quality information can impact Sam's future beliefs as if it were higher-quality (and vice versa)
- overconfidence after low-quality information, under confidence after high-quality information
- eg. jurors are swayed by the testimony of a later discredited witness
- eg. overconfident beliefs of readers of heavily biased or low-quality news sources
- Why are there still so many anti-vaxxers, longer after Wakefield was discredited?
- eg. Why do so many Republicans disbelieve in Global Warming?

Plan for Today's Talk

- I focus on the "time series" aspects of source amnesia, offering it as a unified explanation for
  1. *non-martingale beliefs*
  2. *excess volatility*
  3. *rational bubbles and crashes*
- then some behavioral implications of "cross-sectional" source amnesia

Basic Model

- two equally likely states of the world, $\theta = G, B$ (constant for now)
- an *information source* is a garbling of a "mother" signal:
    - initial realization (the "mother signal") is either a "good" source g or a "bad" source b, and matches the state with chance q
    - each time the source is resampled later, it may be garbled:
        - it repeats the mother signal realization with chance $1 - \varepsilon$
        - flips it with chance ε

Source Sampling Process

- With time series source amnesia, old information sources can be repeatedly sampled according to a natural sampling process.
- Each day, a new source is generated with chance λ.
- any source generated on day τ is called "source τ"
- each day t, Sam the source amnesiac decision theorist (or the current trader, Samantha) consults an information source according to a geometric sampling process: if $\lambda = 1$,

$$\Pr(\text{sample source } \tau) = \frac{\delta^{t+1-\tau}}{1 + \delta + \cdots + \delta^t}$$

where $\delta \in [0, 1]$

The Importance of Mother Signal Garbling

- if $\varepsilon = 0$ (no garbling), then all that matters is *whether* he's consulted a source before:
  - good sources generate g's, and bad sources generate b's
- if $\varepsilon > 0$, then an information source can generate g's and b's, and so repeatedly sampling from it is informative
  - the inference about θ intuitively depends on the *net* $\#$ g's observed from each source

(Weak) Source Amnesia for Sam

1. Sam the DM recalls *all past signal realizations*, but not their information sources
2. Each period t, Sam sees the date stamp $\tau \in \{1, 2, ..., t\}$ on the source he consults, and updates his belief $\mu_t = \Pr(\theta = G)$
3. At the start of each day t, Sam **recalls his current belief** μ_{t-1} but **not** his full history of beliefs
4. Sam understands the source sampling process and his source amnesia, and makes all possible inferences using Bayes' rule

Strong Source Amnesia for Sam

- Here, Sam does not see or forgets the source time stamp

(Weak) Source Amnesia for Samantha

- each day t, a myopic trader (the day-t Samantha) sees:
  - all transactions made by past Samanthas
  - a signal realization (g or b), and its source $\tau \in \{1, 2, ..., t\}$
  - the **current market price** p_{t-1}
- the market price p_t is updated based on the last price p_{t-1} *and reflects all of the period-t Samantha's information*
  - black-box "no-regret" price process like Glosten-Milgrom, which essentially assumes that Samantha trades differently depending on her beliefs about her signal's freshness, and that this info is fully incorporated into the price

Strong Source Amnesia for Samantha

- restrict each Samantha to buy or sell one unit
- then, any Samantha with a good signal (regardless of freshness) buys, and any Samantha with a bad signal (regardless of freshness) sells
- information about her source's timestamp is unobservable: the market sees (and updates the price) based only on whether she has g or b

Focus of Talk

- Henceforth, I largely focus on Sam the behavioral decision maker.
- But each result for Sam has an equivalent counterpart in the market setting for the rational trader Samantha.

An Example of Weak Source Amnesia

- Suppose Sam sees g's from source 1 on days 1 and 2.
- He knows on day 2 that his g is from a stale source, and so his belief entering day 3 reflects this.
- But now, suppose on that on day 3, he sees g from source 2 or 3: either way, he is certain that it is fresh information, and updates his beliefs accordingly.
- Entering day 4, he only recalls: (i) he's seen three g's; and (ii) his current belief μ_3, which indicates that two g's came from the same source, one from an independent source
- but actually *four source histories* that could have led to this belief: $\{112, 113, 121, 122\}$: Sam believes all are possible

Reformulation as a Hidden-State Markov Process

- For inferences about the state $\theta \in \{G, B\}$, a sufficient statistic is the *net # g's observed from each source*
- On each day t, define the *information state* $\gamma = (\gamma_1, \gamma_2, ..., \gamma_t)$ as the vector specifying the net # g's observed from each source
  - $\gamma_\tau = \#g$'s - $\#b$'s from source τ
- The information state evolves according to a Markov process, but is typically hidden from Sam.
- Let ν_t denote Sam's beliefs about the information state.
  - this is a probability measure, which depends on his current belief μ_{t-1} about θ, and on his recollection of the past signal realizations

Example, Continued

- On day 4, Sam's belief + recalled signal realizations reveal that he has seen three g's, from one of the source sequences $\{112, 113, 121, 122\}$
- if $\lambda = 1$ (so a new source is always generated), source sequence 112 has the same sampling chance as 121, and each is sampled δ times as often as 113 or 122
- Associated information states:

$$\begin{aligned} ggg \text{ from } 112 \text{ or } 121 \quad &\Rightarrow \quad \gamma = (2,1,0) \\ ggg \text{ from } 113 \quad &\Rightarrow \quad \gamma = (2,0,1) \\ ggg \text{ from } 122 \quad &\Rightarrow \quad \gamma = (1,2,0) \end{aligned}$$

- So on day 4, $\nu_4(2,1,0) = \frac{2\delta}{2\delta+2}$, $\nu_4(2,0,1) = \frac{1}{2\delta+2}$, and $\nu_4(1,2,0) = \frac{1}{2\delta+2}$

Non-Martingale Beliefs/Foundation for Technical Analysis

- Consider the perspective of an observer who has *kept track of the full sequence of past beliefs (or prices, in a market setting)*
- In our running example:
  - if the *ggg* came from sources $\{112, 113\}$, the observer will know this, from his memory of the day-2 belief μ_2
  - if the *ggg* came from sources $\{121, 122\}$, the observer will know this, from his memory of the day-2 belief $\mu_2' \equiv \mu_2$
- Our first result will assert that if an observer tracks his past beliefs, then he expects *systematic drift in Sam's belief*

A Foundation for Technical Analysis

- Typically, in rational Bayesian models, the current price of an asset reflects all available information (Markov), and is a martingale:
  - If information sets are nested, then the law of iterated expectations implies that today's price is the best forecast of tomorrow's price
- A "technical analyst" is often mocked by financial theorists, since he seeks trading insights from past prices
- But given source amnesia, there is uncertainty about how many times each piece of information has already been traded upon

⇒ information sets are **no longer nested:** information today can be lost tomorrow

- Samantha the technical analyst may correctly expect the price to drift

Continuing Our Example

- Assume the limit case $\lambda = 1$ (a new source is generated each period), $\delta = 1$ (all past sources equally likely to be sampled), and $\varepsilon = 0$ (no garbling of the mother signal)
- Sam's belief about the information state puts chance $\frac{1}{2}$ on information state $(2, 1, 0)$, chance $\frac{1}{4}$ on information state $(2, 0, 1)$, and chance $\frac{1}{4}$ on information state $(1, 2, 0)$.
- But an observer who recalls μ_2 has different beliefs:
  - If Sam consulted sources 112 or 113, the observer puts probability $\frac{1}{2}$ on information state $(2, 1, 0)$, and chance $\frac{1}{2}$ on information state $(2, 0, 1)$
  - If Sam consulted sources 121 or 122, the observer puts probability $\frac{1}{2}$ on information state $(2, 1, 0)$, and chance $\frac{1}{2}$ on information state $(1, 2, 0)$

Analyzing Our Example

- Suppose that Sam consulted the source sequence 113
- If (on day 4) he consults source 1 or 4, or sees *b* from source 2, then he reacts correctly

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- If he sees g from source 2, then his posterior on G is *too low*:
  - He thinks it fresh with chance $\frac{1}{4} = \frac{\Pr(113)}{\Pr(112,121,122,113)}$ while the observer thinks it is fresh with chance $\frac{1}{2} = \frac{\Pr(113)}{\Pr(112 \text{ or } 113)}$

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- If he sees g from source 3, then his posterior on G is *too high:*
  - He thinks it fresh with chance $\frac{3}{4} = \frac{\Pr(112,121,122)}{\Pr(112,121,122,113)}$, while the observer it is fresh with chance $\frac{1}{2} = \frac{\Pr(112)}{\Pr(112 \text{ or } 113)}$

Analyzing Our Example

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- If he sees g from source 3, then his posterior on G is *too high:*
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- The posterior is convex in "freshness", the overreaction to a g from source 3 exceeds the underreaction to a g from source 2
- An observer thinks g from source 3 is as likely as g from source 2, so on balance, expects Sam's posterior is too high tomorrow

Analyzing Our Example

- An outside observer expects Sam's posterior to drift up because Sam *underestimates the freshness of his good information*
  - Source 3 is "fresher" (newer + lower chance of having already been seen) than source 2, but Sam puts chance $\left(\frac{3}{4}, \frac{1}{4}\right)$ on having seen sources (2,3), compared to the observer's probabilities $\left(\frac{1}{2}, \frac{1}{2}\right)$
  - Thus, Sam's beliefs about which sources he has already seen shift probability from fresher to less-fresh sources
- Similarly, if the observer knows $\{112, 122\}$, then Sam comparatively *overestimates the freshess of his good information,* and the observer expects downward drift in Sam's posterior

Formal Analysis

- Let Δ_τ^ν denote the *potential impact of source* τ on Sam's belief, given a probability distribution ν over information states.
- Specifically, Δ_τ^ν is the chance of sampling source τ tomorrow, times the gap between Sam's posteriors if he sees g vs b from source τ:

$$\Delta_\tau^\nu = \Pr(\sigma_{t+1} = \tau) \cdot (\Pr(G|\nu, g \text{ from } \tau) - \Pr(G|\nu, b \text{ from } \tau))$$

  where Sam samples source τ tomorrow with chance $\Pr(\sigma_{t+1} = \tau)$
- Let Y_τ^ν be the chance that the next observation from source τ is g, given the measure ν over information states

Sam's Belief Drift

- Consider the perspective of an observer whose belief about the information state is ν' (based on full recollection of Sam's past posteriors), but who knows that Sam's belief is ν (based on the current posterior)
- Given ν', the observer's expected drift $D_t(\nu')$ in Sam's posterior on G is:

$$D_t(\nu') = E[\mu_{t+1}|\nu'] - \mu_t$$

- This can be written as:

$$D_t(\nu') = \sum_{\tau=1}^{t+1} \left(Y_\tau^{\nu'} - Y_\tau^{\nu} \right) \Delta_\tau^{\nu} \tag{1}$$

A Partial Order of Informational Freshness

Definition

Fix a belief ν by Sam about the information state. Then source τ is fresher than source τ' if source τ has a larger potential impact, i.e. if $\Delta_{\tau}^{\nu} > \Delta_{\tau'}^{\nu}$.

Sufficient Conditions for Freshness: Correct Beliefs

- We will treat the incorrect case separately.
- If the observer thinks Sam's posterior $\Pr(\theta = G)$ is *correct*, then:

♣ in the limit $\delta \to 1$ and $\lambda \to 1$: source τ is fresher than source τ' if Sam thinks he has sampled source τ' more often than source τ, in a FOSD sense

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♠ in the limit $\delta \to 1, \lambda \to 1$, and $\varepsilon \to 0$:

- Let $p^{\nu}_{\tau}(0)$ be the chance (under Sam's belief ν) that Sam has seen 0 net g's from source τ
- Source τ is fresher than source τ' if $p^{\nu}_{\tau}(0) > p^{\nu}_{\tau'}(0)$

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  - Source τ is fresher than source τ' if $p^{\nu}_{\tau}(0) > p^{\nu}_{\tau'}(0)$

- if $\delta < \min\{\mu_t q + (1-\mu_t)(1-q), \mu_t(1-q) + (1-\mu_t)q\}$, then source τ is fresher than source τ' if $\tau > \tau'$

Over/Underestimating Freshness

Some immediate observations from (1):

- the observer expects *upward drift* if $Y_\tau^{\nu'}$ is skewed towards fresher sources, compared to Y_τ^{ν}
- that is, if Sam underestimates the chance of seeing a g tomorrow from the fresher sources (and instead overestimates the chance of a g from less-fresh sources), then the observer expects Sam's posterior on G to drift up
- but the chance of seeing a g tomorrow from source τ is strictly increasing in the # g's already seen from source τ
- so, an observer expects Sam's posterior to drift up if he knows that Sam underestimates the # g's he's already seen from the freshest sources (or overestimates the # b's)
- similarly for downward drift

Limit Result

- In general, the conditions that guarantee upward/downward drift are strong.
- So, consider the limit as $\lambda \to 1$ and $\varepsilon \to 0$ (a new noiseless source is generated each day)
- $\varepsilon = 0$ means that for making inferences about $\theta \in \{G, B\}$, all that matters to Sam is whether he has *ever* seen a source before:
  - let Q_τ^ν be the chance (under Sam's belief ν) that Sam has seen at least one b from source τ
  - let P_τ^ν be the chance that Sam has seen at least one g from source τ
  - and let $p_\tau^\nu(0)$ be the chance that he's never consulted source τ

- useful result: because of the way that the information sets "pool", it turns out that whenever Sam and the observer hold different beliefs about what Sam has already seen from source τ, either
  - $Q_\tau^\nu = Q_\tau^{\nu'} = 0$, i.e. they both think source τ is either unsampled, or a g-source
  - or $P_\tau^\nu = P_\tau^{\nu'} = 0$, i.e. they both think source τ is either unsampled, or a b-source
- let G^ν be the set of all sources for which $p_\tau^\nu(0) \neq p_\tau^{\nu'}(0)$, and $Q_\tau^\nu = Q_\tau^{\nu'} = 0$; and (re)label the sources in G^ν from least-to-most fresh (g_1 thru g_m)
- let B^ν be the set of all sources for which $p_\tau^\nu(0) \neq p_\tau^{\nu'}(0)$, and $P_\tau^\nu = P_\tau^{\nu'} = 0$; and (re)label the sources in B^ν from least-to-most fresh (b_1 thru b_n)

- recall: for $\varepsilon \to 0$ and correct beliefs about θ, source τ is fresher than τ' if either (i) δ is low and τ is chronologically newer than source τ', or (ii) δ is high, and Sam puts a higher chance on having already sampled source τ' than source τ

Partial Orders on Beliefs about Freshness

Definition

Sam underestimates the freshness of his good information if $(P^{\nu'}_{g_1}, P^{\nu'}_{g_2}, \ldots, P^{\nu'}_{g_m}) \succ_{\mathsf{FOSD}} (P^{\nu}_{g_1}, P^{\nu}_{g_2}, \ldots, P^{\nu}_{g_m})$

Definition

Sam overestimates the freshness of his bad information if $(Q^{\nu}_{b_1}, Q^{\nu}_{b_2}, \ldots, Q^{\nu}_{b_n}) \succ_{\mathsf{FOSD}} (Q^{\nu'}_{b_1}, Q^{\nu'}_{b_2}, \ldots, Q^{\nu'}_{b_n})$

In words: Sam overestimates the freshness of his good information if for each unsampled-or-g-source τ, the observer (belief ν') thinks he's seen more g's from sources τ or fresher than Sam (belief ν) does.

General Limit Result

We focus on the case when new sources are nearly always generated ($\lambda \to 1$) and mother signals are rarely garbled ($\varepsilon \to 0$)

Theorem

In the limit as $\lambda \to 1$ and $\varepsilon \to 0$:

1. *If the observer thinks Sam's posterior μ_t on state G is correct, then he expects upward (downward) drift in μ_t if Sam underestimates (overestimates) the freshness of his good information, or overestimates (underestimates) the freshness of his bad information.*
2. *(Self-correcting errors): If the observer thinks Sam's posterior μ_t on state G is too high (too low), then he expects downward (upward) drift in μ_t*

Proof Musings:

- The second result is easier to show: the observer expects upward drift whenever he puts a higher chance than Sam does on seeing a g from the freshest sources tomorrow.
- But for the freshest, least-likely-to-have-been seen sources, the chance of seeing g tomorrow is strictly increasing in $\mu_t \equiv \Pr(\theta = G)$, so Sam underestimates it if his posterior is too low.

In Practice: When Does This Happen?

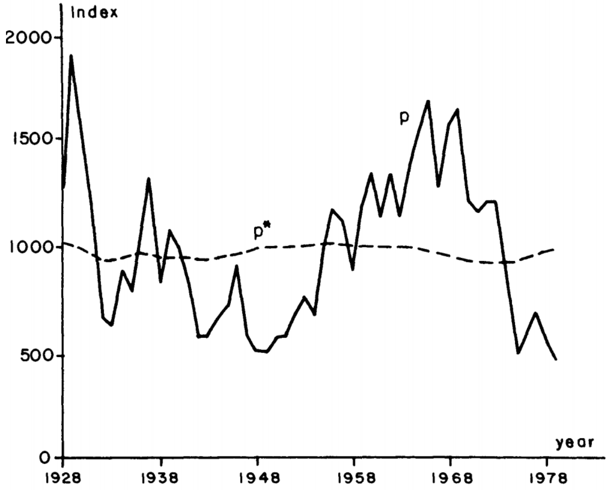
- if Sam has seen *some* stale information (revealed by μ_t) prior to day t, but then sees fresh information on day t, he will forget by tomorrow that he saw a fresh observation on day t: old + new pools with new + old for Sam
- but the observer *will* remember that Sam saw a fresh observation on day t
- so this is a case where Sam underestimates the chance that he has already seen an observation from the freshest source, t

Extrapolating

- if the observer thinks μ_t is correct, he expects upward drift if Sam sees a fresh g (or an old b) on day t, and downward drift if Sam sees an old g (or a fresh b) on day t
- if the observer thinks μ_t is incorrect, he expects it to drift in the direction of the truth tomorrow

Excess Price Volatility

Shiller's 1981 Plot of Dow Jones vs. Ex Post 'Rational' Price



How Source Amnesia Increases Belief and Price Volatility

- Source amnesia increases the variance of the price process
- eg. an asset's fundamental value is either 0 (state B) or 1 (state G)
- Let the state fluctuate, according to a Markov process
- Currently, $\Pr(\theta_{t+1} = 1) = \frac{1}{2}$
- Then
$$Var\left(\theta_{t+1}\right) = \frac{1}{2}(1)^2 + \frac{1}{2}(0)^2 - \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

How Source Amnesia Increases Belief and Price Volatility

- but if the asset price on day t equals the expected value based on a signal, say g or b (matches the state with chance q), then:

$$\begin{aligned} Var(P_{t+1}) &= \frac{1}{2}q^2 + \frac{1}{2}(1-q)^2 - \left(\frac{q+1-q}{2}\right)^2 \\ &= Var(\theta_{t+1}) - q(1-q) \end{aligned}$$

- The extra information reduces the variance of P_{t+1} below that of θ_{t+1}
- but if the signal realization (whether g or b) were forgotten by the time prices were set, then $Var(P_{t+1})$ would shoot back up

- Sam's case is not this extreme: he does not mix up g's and b's
- but he does mix up source histories, which increases variance in the price process for the same reason

Increased Volatility: Illustration

- if a price process is followed for 3 periods, with values P_1, P_2, P_3, then the realized variance is

$$\sum_{t=1}^{3} \frac{1}{3}\left(P_t - \frac{P_1+P_2+P_3}{3}\right)^2$$
$$= \frac{2}{9}\left[(P_2-P_1)^2 + (P_3-P_2)^2 + (P_2-P_1)(P_3-P_2)\right]$$

- if a price process is a random walk, then price increments are expected to be uncorrelated
- but for Sam, the price increments are *negatively correlated,* on average: if he expects upward drift today, he expects downward drift tomorrow
- in general, the variance understates the squared price fluctuations

Example

Consider the basic model, with $q = .7$ and $\varepsilon = .1$, and suppose Sam sees *ggggb* from sources 11333 :

t	μ_t^{Sam}	$\mu_t^{\text{non-amnesiac}}$	sources
3	0.815 7	0.8157	$\frac{1}{2}(2,1,0)+\frac{1}{4}(1,2,0)+\frac{1}{4}(2,0,1)^*$
4	0.877 6	0.838 7	$\frac{1}{2}(2,1,1)+\frac{1}{4}(1,2,1)+\frac{1}{4}(2,0,2)^*$
5	.8157	.8157	$\frac{1}{2}(2,1,0)+\frac{1}{4}(1,2,0)+\frac{1}{4}(2,0,1)^*$

- Clearly, source amnesia increases $\sum_{t=3}^{4}(\mu_{t+1} - \mu_t)^2$ in this example
- Reason: Sam's posterior is too high after the 4th-period g (source 3 has already been sampled once, but he thinks it has a 75% chance of being fresh)
- But after a subsequent b from source 3, they adjust to where they should be
- this is one example (there are other realizations of the sampling process for which source amnesia reduces price fluctuations), but the average effect is an increase

Strong Source Amnesia

- so far, we have assumed fairly heroic Bayesian updating: Sam knows his current posterior, figures out all possible histories that could have led to this posterior, sees the timestamp on today's info, and computes exactly the chance that he has already seen the information
- but what about a simpler story, where *all source information is lost?*

Application: Myths (and Bubbles)

- consider again the environment where a new source is generated each period with chance λ
- for simplicity, assume no garbling in the sources: a g-source always says g, and a b-source always says b
- but assume that while Sam keeps track of all the full history of signal realizations, source information is already lost by the time he updates his beliefs

- then, as in the motivating example: if Sam sees a g today, the best he can do is very crudely estimate the chance that it's from an already-sampled source
- his beliefs will be a weighted average of what they should be if it's fresh, and what they should be if it's stale
- but the probability with which he thinks the information is fresh is:
  - increasing in λ (the rate at which new sources are generated)
  - decreasing in the "consistency" of his info so far: react more to a g if you've seen mostly b's, than if you've seen mostly g's

- then in phases when new information is being generated more slowly than expected, Sam's λ is too high, so he *overestimates* the chance that the new information is fresh
- this can lead him to believe in a myth, based on very little info:
    - suppose he believes that new studies on "vaccines cause autism" come out fairly frequently (so λ is high)
    - but in fact, there is just one study
    - then according to our sampling process, Sam will keep seeing this study over and over again, and will keep reacting as if he were seeing a study with a high chance of being new (though gradually, will react less)
    - as long as he sees no information to the contrary, he can develop a high level of confidence in the myth

- similarly, this might be one rational story of why bubbles form (and then collapse)
- if there is actually very little info out there, then the same piece (say from a g-source) will keep showing up
- if the Samanthas believe there is more new information than there is, they will keep (over-)reacting to this g-signal, inflating the perceived value of the asset
- but the inflation happens at a diminishing rate, so any new b-signals (which the Samanthas will realize are new) can rapidly deflate the value

Cross-Sectional Source Amnesia

- consider a world where there are two information sources, each of which provides a conditionally iid signal every period
- however,
  - one source is high-quality, and one source is low-quality
  - or, one source is informative about the value of a random variable X (e.g. Apple hardware value), the other about a random variable Y (e.g. Apple software value), and the individual cares about $E[\alpha X + Y]$, $\alpha \in \mathbb{R}^+$
  - or one source is biased (e.g., always reports g's truthfully, but misreports b's as g's with chance ε)

Cheap Behavioral Biases

- even if Sam recognizes at the time that his information is from a low-quality or biased source, source amnesia may lead him to lose track of the source
  - could this explain why many people select heavily biased news sources (my crazy aunt likes "Natural News"), but fail to fully "de-bias"?
- confirmation bias

Questions and Applications

- informational herding?
- forward-looking Sam: to what extent might he take actions that incur an immediate payoff loss, in order to signal some source info to his future self?
- in the hardware-software story: strong source amnesia will prevent Sam from learning the true values of X and Y, he can at best learn $\alpha X + Y$: but can he always even learn this?
- strategic Samanthas: our traders (the Samanthas) were myopic, each entering the market only once. But a Samantha who sees a new b (or old g) *herself* expects the price to fall tomorrow, implying an incentive to short-sell if she is forward-looking

Conclusions and Applications

- source amnesia is well-documented in the psychology and neuroscience literatures, but we believe it is new to economics
- implications:
  - volatility
  - non-martingale beliefs/prices
  - failure to de-bias
  - bubbles
  - may provide one story for why advertising – which typically sends out the same (very limited) info many times – can be useful

THANK YOU!